



Properties of Stock Options

Chapter 9

Notation

- c : European call option price
- p : European put option price
- S_0 : Stock price today
- K : Strike price
- T : Life of option
- σ : Volatility of stock price
- C : American Call option price
- P : American Put option price
- S_T : Stock price at option maturity
- D : Present value of dividends during option's life
- r : Risk-free rate for maturity T with cont comp

Effect of Variables on Option Pricing

(Table 9.1, page 202)

Variable	c	p	C	P
S_0	+	-	+	-
K	-	+	-	+
T	?	?	+	+
σ	+	+	+	+
r	+	-	+	-
D	-	+	-	+

American vs European Options

An American option is worth at least as much as the corresponding European option

$$C \geq c$$

$$P \geq p$$

Calls: An Arbitrage Opportunity?

- Suppose that

$$c = 3$$

$$T = 1$$

$$K = 18$$

$$S_0 = 20$$

$$r = 10\%$$

$$D = 0$$

- Is there an arbitrage opportunity?

Lower Bound for European Call Option Prices; No Dividends (Equation 9.1, page 207)

$$c \geq \max(S_0 - Ke^{-rT}, 0)$$

Puts: An Arbitrage Opportunity?

- Suppose that

$$p = 1$$

$$S_0 = 37$$

$$T = 0.5$$

$$r = 5\%$$

$$K = 40$$

$$D = 0$$

- Is there an arbitrage opportunity?

Lower Bound for European Put Prices; No Dividends

(Equation 9.2, page 208)

$$p \geq \max(Ke^{-rT} - S_0, 0)$$

Put-Call Parity; No Dividends

(Equation 9.3, page 208)

- Consider the following 2 portfolios:
 - Portfolio A: European call on a stock + PV of the strike price in cash
 - Portfolio C: European put on the stock + the stock
- Both are worth $\max(S_T, K)$ at the maturity of the options
- They must therefore be worth the same today.
This means that $c + Ke^{-rT} = p + S_0$

Arbitrage Opportunities

- Suppose that

$$c = 3 \quad S_0 = 31$$

$$T = 0.25 \quad r = 10\%$$

$$K = 30 \quad D = 0$$

- What are the arbitrage possibilities when $p = 2.25$? $p = 1$?

Early Exercise

- Usually there is some chance that an American option will be exercised early
- An exception is an American call on a non-dividend paying stock
- This should never be exercised early

An Extreme Situation

- For an American call option:
 $S_0 = 100$; $T = 0.25$; $K = 60$; $D = 0$
Should you exercise immediately?
- What should you do if
 - you want to hold the stock for the next 3 months?
 - you do not feel that the stock is worth holding for the next 3 months?

Reasons For Not Exercising a Call Early (No Dividends)

- No income is sacrificed
- Payment of the strike price is delayed
- Holding the call provides insurance against stock price falling below strike price

Should Puts Be Exercised Early ?

Are there any advantages to exercising an American put when

$$S_0 = 60; T = 0.25; r = 10\%$$

$$K = 100; D = 0$$

The Impact of Dividends on Lower Bounds to Option Prices

(Equations 9.5 and 9.6, pages 214-215)

$$c \geq S_0 - D - Ke^{-rT}$$

$$p \geq D + Ke^{-rT} - S_0$$

Extensions of Put-Call Parity

- American options; $D = 0$

$$S_0 - K < C - P < S_0 - Ke^{-rT}$$

(Equation 9.4, p. 211)

- European options; $D > 0$

$$c + D + Ke^{-rT} = p + S_0$$

(Equation 9.7, p. 215)

- American options; $D > 0$

$$S_0 - D - K < C - P < S_0 - Ke^{-rT}$$

(Equation 9.8, p. 215)